Domain-Wall Waves (2D Magnons) in Superconducting Ferromagnets

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Propagation of the magnetization waves along domain walls (2D magnons) in a superconducting ferromagnet has been studied theoretically. The magnetostatic fields (long-range dipole-dipole interaction) have a crucial effect on the spectrum of 2D magnons. But this effect is essentially affected by the superconducting Meissner currents, which screen the magnetostatic fields and modify the long-wavelength spectrum from square-root to linear. The excitation of the domain wall waves by an electromagnetic wave incident on a superconducting-ferromagnet sample has been considered. This suggests using measurements of the surface impedance for studying the domain wall waves, and eventually for effective probing of superconductivity-ferromagnetism coexistence.

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Coexistence of superconductivity and ferromagnetism, which results in a number of unusual phenomena, has already been studied about 50 years [1]. A revival of research in this area was stimulated by experimental observation of superconductivity-ferromagnetism coexistence in high-T_c superconductors [2, 3] and various unconventional superconductors [4, 5, 6, 7]. An essential obstacle for experimental detection of superconductivity-ferromagnetism coexistence is screening of internal magnetic fields generated by the ferromagnetic order parameter (spontaneous magnetization) by superconducting currents. A possible way to overcome this obstacle is to investigate spin waves, which are a direct evidence of the presence of spontaneous magnetization [8, 9]. They can be excited by the electromagnetic (EM) wave incident on a superconducting ferromagnet (SCFM).

In Ref. 9 spin waves in SCFMs have been studied for the Meissner state with the uniform spontaneous magnetization, i.e., for a single-domain sample. Though at the equilibrium superconductivity suppresses usual ferromagnetic domain structures with periods determined by demagnetization factors and sizes of samples [10], the interplay with superconductivity can lead to formation of intrinsic domains of the order of the London penetration depth or less [11] (see the latest discussion in Ref. 12). Moreover, a domain wall (DW) is a topologically stable defect, and the presence (absence) of DWs can be determined by the sample prehistory but not by the conditions of the equilibrium. This justifies an interest to studying spin waves propagating along DWs (2D magnons).

In normal ferromagnet (FM) the DW dynamics and DW waves have already been studied a few decades and are important for various applications [13]. The crucial feature of 2D magnons is that the magnetostatic effects (dipole-dipole interaction) are much more important for them than for 3D bulk magnons. On the other hand one may expect that SC should influence these effects first of all since it prohibits penetration of the magnetostatic fields deep into the bulk of domains.

In the present Letter we investigate theoretically DW waves in a SCFM. The analysis confirms expectation of strong effect of superconductivity on DW waves: Meissner screening of magnetostatic fields in domains modifies the square-root 2D-magnon spectrum revealed for normal FM [13] to a linear sound-like spectrum. We show how the 2D magnons can exhibit themselves in the linear response to the AC magnetic field (surface impedance). As well as in the case of bulk 3D spin waves, detection of 2D magnons, being interesting itself, would provide a direct probe of superconductivity-ferromagnetism coexistence. The spectrum of 2D magnons is sensitive to bulk and surface pining of DWs, which can lead to a gap in the spectrum and a peak in the EM wave absorption.

Let us consider a 180° DW in a FM with magnetic anisotropy of the easy-axis type [Fig. 1(a)]. The z axis is the easy axis and the DW is parallel to the xz plane and separates two domains with spontaneous magnetization $\mathbf{M}$ along the $+z$ ($y < 0$) and $-z$ ($y > 0$) directions. Neglecting the magnetostatic fields, the structure of the DW is well known [13, 14]. Rotation of the magnetization $\mathbf{M} = M_0(\cos\phi\ \sin\theta, \sin\phi\ \sin\theta, \cos\theta)$ inside the DW is described in polar and azimuthal angles θ and ϕ as [14]

$$\theta_0 = 2\tan^{-1} e^{y/\Delta} . \qquad (1)$$

Here $\Delta = (A/K)^{1/2}M_0$ is the DW width, A is the constant, which determines the energy of the non-uniform exchange $A\nabla_j\mathbf{M}\cdot\nabla_j\mathbf{M}$, and the constant K determines the easy-axis anisotropy energy $K\sin^2\theta$. The azimuthal angle ϕ is an arbitrary constant as far as the magnetostatic (dipole) fields are neglected. In normal FMs currents are absent at the equilibrium , and the magnetic field $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$ is curl-free, i.e., is a potential field with sources called "magnetic charges" $\rho_m = -\nabla\cdot\mathbf{M}$: $\nabla\cdot\mathbf{H} = \nabla\cdot\mathbf{B} - 4\pi\nabla\cdot\mathbf{M} = 4\pi\rho_m$. The energy of the magnetostatic fields lifts degeneracy with respect to the angle ϕ: the ground state corresponds to the DW of the Bloch type with $\phi = 0$. This means that the magnetization rotates in the DW plane, and the magnetic charges do not appear ($\nabla\cdot\mathbf{M} = 0$). This structure, which was obtained for normal FMs, remains valid at scales of or-

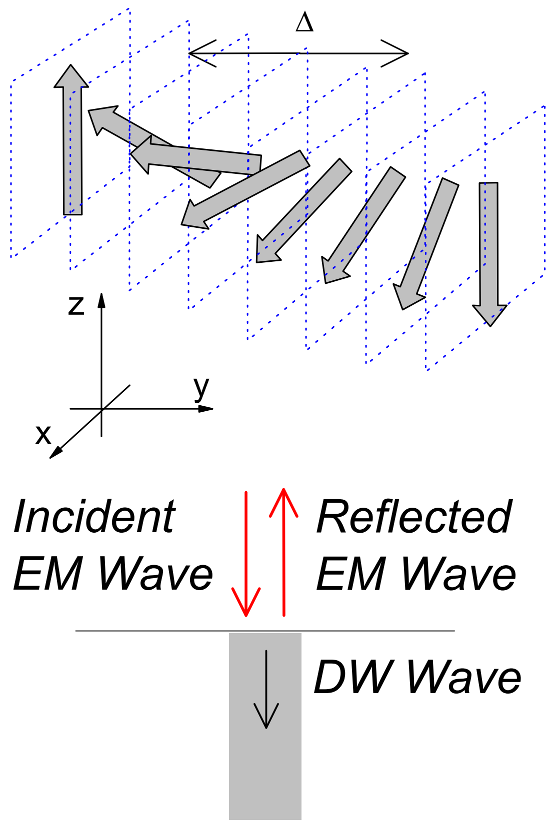

FIG. 1: (Color online) (a) The 180° Bloch domain wall. The magnetizations in the domains are parallel and antiparallel to the z axis. Inside the DW the magnetization rotates in the xz plane. (b) Excitation of DW waves by an incident EM wave.

der Δ also for SCFMs, as far as the London penetration depth λ essentially exceeds the DW thickness Δ. However, the difference between a normal FM and a SCFM is important at distances larger than Δ: while in normal FMs the magnetic field $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$ vanishes and the magnetic induction $\mathbf{B} = 4\pi\mathbf{M}$ is constant inside domains, in SCFMs the magnetic induction $\mathbf{B}$ is confined in the Meisssner layers of width λ [10]:

$$B_z = \pm 4\pi M_0 e^{\pm y/\lambda} \ , \tag{2}$$

where the upper and the lower signs correspond to $y < 0$ and $y > 0$ respectively. Thus the Meissner currents $j_x = -(c/4\pi) dB_z/dy$ screen out the main bulk of domains from the magnetic induction.

We shall use the DW dynamics developed for FMs with high quality factor $\alpha = K/2\pi M_0^2$ [13]. In this limit one may assume that the structure of the Landau-Lifshitz DW, which is given by Eq. (1), is not affected essentially by dynamical processes, and the DW dynamics can be described in the terms of the pair of canonically conjugated variables: the azimuthal angle $\phi(x, z; t)$, which determines the plane of magnetization rotation, and the displacement of the DW $\eta = \eta(x, z; t)$ along the y axis: $\theta(x, y, z; t) = \theta_0(y - \eta(x, z; t))$. Neglecting dissipation, the equations of DW motion (Slonczewski's equations [15]) are the Hamilton equations:

$$\partial_t \eta = \frac{\gamma}{2M_0} \frac{\delta H}{\delta \phi} \ , \quad \partial_t \phi = -\frac{\gamma}{2M_0} \frac{\delta H}{\delta \eta}, \tag{3}$$

where $\delta/\delta\phi$ and $\delta/\delta\eta$ are functional derivatives and the Hamiltonian $H = \int \sigma dx\, dz$ is determined by the surface energy density σ of the DW. The Hamilton equations (3) point out a direct analogy of $(2M_0/\gamma)\phi$ with a canonical momentum conjugate to "coordinate" η. But it is worthwhile to mention another analogy. The displacement η leads to the change $2M_0\eta$ of the z-component of the total magnetization per unit area of the DW. Since ϕ is the angle of spin rotation around the z axis, Eqs. (3) can be also considered as Hamilton equations for the pair of conjugate variables "angular momentum–angle".

The DW surface energy is

$$\sigma(\phi, \eta) = 4\pi M_0^2 \Delta\alpha \left[(\nabla\eta)^2 + \Delta^2(\nabla\phi)^2\right] + \frac{1}{8\pi}\int dy(\mathbf{h}^2 + \lambda^2[\nabla \times \mathbf{h}]^2)$$
$$+\frac{1}{4\pi}\int dy(\mathbf{H}^{(0)} \cdot \mathbf{h}_2 + \lambda^2[\nabla \times \mathbf{H}^{(0)}] \cdot [\nabla \times \mathbf{h}_2]) \ . \tag{4}$$

The terms $\propto \alpha$ (the first square brackets) present the contributions of the exchange energy and the anisotropy energy, whereas the integral terms are the energies of superconducting screening currents and of the magnetostatic fields $\mathbf{H} = \mathbf{H}^{(0)} + \mathbf{h} + \mathbf{h}_2$. Here $\mathbf{H}^{(0)}$ is the magnetic field in the ground state, and $\mathbf{h}$ and $\mathbf{h}_2$ are the dynamic corrections to the field of the first and the second order in the wave amplitude respectively. Necessity to take into account the second-order corrections is a peculiar feature of the DW dynamics in SCFM: in normal FM $\mathbf{H}^{(0)} = 0$ and these corrections vanish.

The first-order contribution to the magnetic induction $\mathbf{b} = \mathbf{B} - \mathbf{B}^{(0)} = \mathbf{h} + 4\pi\mathbf{m}$ can be found from the generalized London equation:

$$-\Delta\ \mathbf{b} + \lambda^{-2}\mathbf{b} = 4\pi\nabla \times \nabla \times \mathbf{m}. \tag{5}$$

The dynamic components of the magnetization $\mathbf{m} = \mathbf{M} - \mathbf{M}^{(0)}$ are defined through the conjugated variables η and ϕ as follows:

$$\mathbf{m} = M_0 \begin{pmatrix} -\eta\, d[\sin\theta_0(y)]/dy \\ \phi \sin\theta_0(y) \\ -(\eta/\Delta)\sin^2\theta_0(y) \end{pmatrix}. \tag{6}$$

Let us consider the plane wave with wave vector $\mathbf{k} = (0, 0, k)$ parallel to the z axis: $\phi, \eta, \mathbf{b} \propto \exp(-i\omega t + ikz)$. Then the London equation (5) becomes a set of ordinary differential equations with the only coordinate y, and its solution is:

$$\mathbf{b} = -\frac{4\pi}{\widetilde{k}}\left[e^{\widetilde{k}y}\int_y^\infty dy' \nabla \times \nabla \times \mathbf{m}(y') e^{-\widetilde{k}y'} + e^{-\widetilde{k}y}\int_{-\infty}^y dy' \nabla \times \nabla \times \mathbf{m}(y') e^{\widetilde{k}y'}\right]. \tag{7}$$

Here $\widetilde{k}^2 = k^2 + \lambda^{-2}$.

Now one can substitute the magnetic field $\mathbf{h} = \mathbf{b} - 4\pi\mathbf{m}$ into the first magnetostatic integral in Eq. (4) and perform integration. The contribution of the second-order corrections $\mathbf{h}_2$ [the second magnetostatic integral in Eq. (4)]can be determined from general arguments. The second-order terms contain the Fourier components of η^2 with zero wave vector and with the wave vector $2k$ (the second harmonic). Only the first component, which is z-independent, can interfere with the zero-order terms. This yields the k-independent contribution to the surface energy, which should provide translational invariance: the surface energy should not depend on the spatially independent DW displacement. Altogether this yields the following surface energy density in variables η and ϕ:

$$\sigma = 4\pi M_0^2 \left[\alpha k^2\Delta(\eta^2 + \phi^2\Delta^2) + \left(1 - \frac{\Delta k^2}{|\widetilde{k}|}\right)\phi^2\Delta + \left(|\widetilde{k}| - \frac{1}{\lambda}\right)\eta^2\right] , \tag{8}$$

where the second-order terms are presented by the negative term $-\eta^2/\lambda$. All terms, which are not proportional to the quality factor α, originate from the magnetostatic energy and the SC currents. The magnetostatic term $\propto \phi^2$ plays a role of the "kinetic" energy since the angle ϕ is the momentum conjugate to the displacement η. In the limit $k \to 0$ it is determined by the magnetic fields inside the DW and can be expressed via the Döring mass $m_D = 1/\gamma^2 2\pi\Delta$ [13]. The k-dependent contribution to the kinetic energy represents the energy of the magnetostatic fields outside the DW in the layer of width $\sim 1/|\widetilde{k}|$. Altogether the magnetostatic terms $\propto \phi^2$ are connected with the double layer of magnetic charges, which appears when the DW rotates from the Bloch orientation $\phi = 0$. On the other hand, the magnetostatic term $\propto \eta^2$ is connected with the magnetic charges, which appear due to rotation of the DW with respect to the direction of the magnetization in domains.

Using Eq. (8) in Slonczewski's equations (3) transformed to the Fourier presentation one obtains the spectrum of plane waves propagating along the DW:

$$\omega^2 = \omega_M^2 \left[\alpha k^2\Delta^2 + \frac{\Delta}{\lambda}\left(\sqrt{k^2\lambda^2+1} - 1\right)\right] \times \left(1 + \alpha k^2\Delta^2 - \frac{k^2\lambda\Delta}{\sqrt{k^2\lambda^2+1}}\right), \tag{9}$$

where $\omega_M = 4\pi\gamma M$.

In the limit $\lambda \to \infty$ (no superconductivity), this transforms to the spectrum of 2D magnons in normal FMs:

$$\omega^2 = \omega_M^2(\alpha k^2\Delta^2 + |k|\Delta)(1 + \alpha k^2\Delta^2 - |k|\Delta). \tag{10}$$

All nonanalytic terms $\propto |k|$ in this expression are of magnetostatic origin and are related with penetration of magnetostatic fields deep into domains on scales of the order of the wavelength $2\pi/k$. They lead to the nonanalytic wave spectrum $\omega \propto \sqrt{k}$ in the long-wavelength limit, which was known before [13]. Another nonanalytic term, which appears in the second multiplier with the negative sign, has not been considered so far as far as we are aware.

Returning back to SCFM one can see that the Meissner screening eliminates non-analytic features of the spectrum and allows expansion in k^2. Thus in the long-wavelength limit the spectrum is sound-like:

$$\omega = c_s k, \tag{11}$$

where $c_s = \omega_M\sqrt{\Delta(\alpha\Delta + \lambda/2)}$ is the 2D spin-wave velocity.

In order to find the effect of DW waves on surface impedance one should solve a boundary problem. In general the wave spectrum Eq. (9) leads to differential equations of high order in the configurational space (bearing in mind the correspondence $k \to -i\partial/\partial z$). But the sound-like spectrum Eq. (11) reduces Slonczewski's equations in the configurational space to two ordinary differential equations of the first order:

$$\partial_t\eta = \omega_M\Delta\phi - \gamma\Delta h_{ext} \ , \quad \partial_t\phi = \frac{c_s^2}{\omega_M\Delta}\frac{\partial^2\eta}{\partial z^2} \ . \tag{12}$$

In the first equation we added the interaction with the external magnetic field. This field arises from the EM wave of frequency ω incident on the sample surface and linearly polarized along y-axis [Fig. 1(b)]. The EM wave penetrates through the surface of SCFM at the distance $\sim \lambda$: $h_{ext} = h_0 e^{z/\lambda}$. Here h_0 is the AC magnetic field at the sample surface $z = 0$.

Next is to formulate boundary conditions for these equations. One boundary condition is imposed at $z \to -\infty$. We assume that the DW plane wave, which is excited near the sample surface $z = 0$ is the only propagating wave in the bulk and there is no reflected wave coming from $z = -\infty$. The second boundary condition is imposed at the sample border $z = 0$. Assuming an ideal surface without any surface force the balance of the "momentum" ϕ requires that the DW remains normal to the sample border: $\partial\eta/\partial z = 0$. But the magnetic field h_{ext} generated by the incident EM wave , though being a bulk force, is confined to the Meissner layer of width λ, which is much smaller than the wavelength. Then one may consider its integral effect as that of a surface force. This leads to modification of the boundary condition for propagating wave, which must be written as

$$c_s^2 \frac{\partial\eta}{\partial z}\bigg|_{z=0} - \frac{\pi}{2} i\omega\gamma h_0\Delta\lambda = 0 \ . \tag{13}$$

This boundary condition determines the amplitude of the wave propagating from the sample border to $z = -\infty$:

$$\eta = \frac{\pi}{2}\frac{\omega}{c_s^2 k}\gamma h_0\Delta\lambda \exp\left(-i\omega t - ikz\right) \ . \tag{14}$$

The wave is accompanied by the energy flux, which is determined from the energy balance $\partial_t \sigma + \partial S/\partial z = 0$:

$$S = -\frac{\partial \sigma}{\partial(\partial \eta/\partial z)}\partial_t \eta = -\frac{\pi}{16}\frac{\omega^2}{c_s}\Delta\lambda^2 h_0^2 \ . \quad (15)$$

The energy brought away by the DW plane wave is supplied by the wave incident on the surface: at the reflection of the wave from the sample surface some part of energy is absorbed though no dissipation mechanism is present explicitly in our model: it is assumed that dissipation occurs deep inside the sample where the spin wave eventually dissipates and does not return as a reflected wave. Energy absorption at reflection of the EM wave from the sample surface is characterized by real part of the surface impedance, which is defined as a ratio of the tangential components of electric field e_x and magnetic field h_0 at the surface: $\zeta = e_x/h_0|_{z=0}$. Namely, absorption per unit area per second is given by $(ch_0^2/8\pi)\mathrm{Re}\zeta$. Equating this to the average energy flux per unit area given by $|S|n_W$ one obtains that

$$\mathrm{Re}\zeta = 8\pi n_W \frac{|S|}{ch_e^2} = \frac{\pi^2}{2} n_W \Delta\lambda^2 \frac{\omega^2}{cc_s} \ . \quad (16)$$

Here n_W is the linear density of DWs. However this dependence is essentially modified by bulk and surface pinning of DW. Bulk pinning lifts translational invariance and leads to the gap in the magnon spectrum, which instead of Eq. (11) becomes

$$\omega^2 = \omega_p^2 + c_s^2 k^2 \ . \quad (17)$$

Surface pinning modifies the boundary condition Eq. (13), which now is

$$c_s^2 \left(\frac{\partial \eta}{\partial z} + k_s \eta\right)\bigg|_{z=0} - i\omega\gamma h_0 \Delta\lambda = 0 \ , \quad (18)$$

where k_s is the inverse length characterizing intensity of surface pinning. Repeating the derivation of $\mathrm{Re}\zeta$ in the presence of pinning one obtains that

$$\mathrm{Re}\zeta = \frac{\pi^2}{2} n_W \Delta\lambda^2 \frac{\omega^3\sqrt{\omega^2 - \omega_p^2}}{cc_s(\omega^2 - \omega_p^2 + c_s^2 k_s^2)} \ . \quad (19)$$

Thus in the presence of bulk pinning absorption of the EM wave starts from the threshold frequency ω_p and above the threshold grows as $\sqrt{\omega - \omega_p}$. If at the same time surface pinning is weak enough there is an absorption peak at $\omega - \omega_p \sim c_s k_s$. The peak can correspond to frequencies much lower than the threshold frequency for excitation of bulk spin waves (frequency $\alpha\omega_M$ of the ferromagnetic resonance) [9]. These special features of the absorption spectrum can be useful for experimental detection of DW waves in SCFMs.

Our analysis was based on the classical Landau-Lifshitz dynamics of a spin FM with a single-valued spontaneous magnetization $\mathbf{M}$. Meanwhile in p-wave superconductors superconductivity coexists with *orbital ferromagnetism* when spontaneous magnetic moment is not well-defined and one should develop the magnetic dynamics using the concept of magnetization currents [16]. But as well as in the case of bulk magnons considered in Ref. 16, one may expect that this would lead only to redefinition of the 2D magnon parameters without changing the general picture of their propagation and excitation. We hope to address this problem elsewhere.

In summary, the low-frequency dynamic of the DW in SCFM has been studied and the spectrum of DW waves (2D magnons) has been found. The magnetostatic effects (long-range dipole-dipole interaction) have a crucial influence on the long-wavelength spectrum, while the superconducting Meissner currents modify these effects essentially. They suppress the effect of long range dipole interaction eliminating the non-analytical features of the 2D-magnon spectrum revealed for normal FMs. Our analysis has demonstrated that the response of a SCFM sample to the EM irradiation (surface impedance) provides information on 2D magnons in DWs. Bulk and surface pinning of DWs introduces a gap in the 2D-magnon spectrum and a characteristic peak in the absorption of the incident EM wave (real part of the surface impedance). In conclusion, studying of DW waves promises to be an effective probe of superconductivity-ferromagnetism coexistence.

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